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Abstract

This paper shows how demand-side energy use efficiency affects demand response (DR) and total CO_2 emissions. The marginal cost of DR corresponds to the marginal utility of electricity consumption. Thus, improved energy efficiency increases the marginal cost of DR and increases thermal power generation in balancing markets. We analyze a model consisting of a day-ahead and balancing market and examine CO_2 emissions from each market. Improved energy efficiency decreases emissions from the day-ahead market while increasing emissions from the balancing market. The analysis reveals that improved energy efficiency could increase total emissions when the emission factor of the marginal plant in the day-ahead market is sufficiently small. Raising the carbon tax rate as energy efficiency improves will be necessary to deter such perverse effects, expanding the use of DR in the balancing market.

Keywords: day-ahead market, balancing market, carbon tax, incentive-based demand response, demand-side energy efficiency

JEL classifications: D02, Q41, Q58

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1 Introduction

With the increasing use of renewable energy sources, the ability to respond to supply-side fluctuations in power systems has become increasingly important. In conventional electricity markets, thermal power sources play an important role in maintaining the balance between supply and demand (Qadrdan et al., 2017; Chen et al., 2024). However, to decarbonize a power system, the demand response (DR) needs to be expanded to reduce the reliance on thermal power sources. In recent years, there has been growing interest in designing balancing markets and using demand-side flexibility (Van der Veen and Hakvoort, 2016; Cabot and Villavicencio, 2024).

In this study, we focus on the impact of demand-side energy use efficiency as a significant factor influencing the marginal cost of DR. Improved energy efficiency increases the amount of energy service produced from each kilowatt hour (kWh) of electricity consumed, increasing the marginal cost of DR.

While several studies have suggested policy implications for the synergistic effects of improving energy efficiency and promoting DR as effective measures to reduce CO_2 emissions (Wohlfarth et al., 2020; Dranka et al., 2022), there is no clear theoretical framework outlining the relationship between the two. While there are many studies on demand-side energy efficiency (Abrardi and Cambini, 2015; Chu and Sappington, 2012, 2013; Arimura et al., 2012; Datta, 2019; Wirl, 1995), these studies focus on energy efficiency as a means to promote energy conservation, and no studies clearly show the relationship with DR.

We examine a microeconomic model that consists of a day-ahead market and a balancing market to analyze the impact of demand-side energy efficiency on DR and total CO_2 emissions. In the day-ahead market, the demand and supply schedule for the following day is established on the basis of the forecasted generation from variable renewable energy sources. The balancing market addresses the discrepancies between the actual generation and the forecasted output of variable renewable sources that arise after the day-ahead market has closed. The incentive-based DR and thermal power generation are assumed to be available in the balancing market to restore the supply–demand balance. While improved energy efficiency reduces emissions from the day-ahead market, it also increases the marginal cost of DR, which leads to an increase in thermal power generation in the balancing market. As a result, total CO_2 emissions could increase under certain conditions. This result indicates that improved energy efficiency can increase CO_2 emissions through a mechanism different from the energy rebound effect.[1]

Chao and DePillis (2013) provided a theoretical framework for incentive-based DR. This framework focuses on the issue of so-called baseline inflation, in which consumers increase their DR baselines to obtain more rewards from demand reduction. In their model, however, the price of DR is treated as an exogenous variable, and the balancing market is not explicitly defined. Our study provides a different perspective by focusing on how energy efficiency affects the marginal cost of DR and explicitly incorporating a balancing market in which both DR and thermal power generation are traded.

The remainder of this paper is organized as follows. Section 2 describes the model for the day-ahead market. In Section 3, we formulate the cost of DR as a function of energy efficiency and derive the amount of DR and thermal power generation in the balancing market equilibrium. Section 4 presents the impact of energy efficiency on emissions in the day-ahead and balancing markets and on total emissions. Section 5 shows how energy efficiency affects the carbon tax rate necessary to meet a specific policy target for total CO_2 emissions. Section 6 concludes the paper.

2 Day-ahead market

2.1 Electricity demand

We formulate a service-oriented consumer utility following Wirl (1995). In the service-oriented framework, the energy services produced by electricity consumption generate utility for consumers. The amount of energy service yielded by each unit of electricity depends on the energy efficiency, denoted by the parameter θ, of the utilized equipment. Following Wirl (1995), we formulate the relationship between the amount of energy service s and the electricity consumption q (kWh) as follows.[2]

$$s = \theta q \tag{1}$$

[1] For the energy rebound effect, see, e.g., Sorrell (2009).

[2] Typically, in a day-ahead market, the 24 hours are divided into multiple periods (denoted by subscript t, for example), and participants buy and sell electricity for each period of the following day. For example, the Japan Electric Power Exchange (JEPX) divides 24 hours into 48 periods ($t = 1, ..., 48$), 30 minutes each. However, since the theoretical properties discussed in this paper are equally valid for all periods, we can omit the subscript t without losing any theoretical generality. Therefore, we omit the subscript t throughout to simplify the notation.

The units of energy service s and energy efficiency θ depend on the energy equipment used. For example, in the case of a battery electric vehicle, the distance traveled (km) corresponds to the amount of energy service, and the distance that can be traveled per 1 kWh of electricity (km/kWh) corresponds to the energy efficiency. For a production facility in a factory, the amount of product (e.g., kg) corresponds to the amount of energy service, and the amount that can be produced with 1 kWh of electricity (kg/kWh) corresponds to the energy efficiency.

Let us express the utility for the amount of energy service s as a quadratic function as follows. [3]

$$u(s) = as - \frac{b}{2}s^2 \tag{2}$$

Note that the generality of the discussion in this paper is not lost by specifying the functional form in this way.

The consumer (which we assume a demand-side aggregator that seeks to maximize consumer surplus on behalf of consumer interests) purchases the electricity to enjoy the energy service s at price p. The utility produced by the energy service minus the expenditure on the purchase of electricity is the consumer surplus in the day-ahead market. The consumer determines the electricity demand $q = q(p; \theta)$ to maximize consumer surplus, taking the electricity price p and energy efficiency θ as given.

$$\underset{q}{\text{maximize}} \; u(s) - pq \tag{3}$$

The first-order condition for the problem of Eq. (3) is as follows:

$$a\theta - b\theta^2 q = p \tag{4}$$

The left-hand side of the Eq. (4) is the marginal utility of electricity consumption, and the right-hand side is the price of electricity. From Eq. (4), we obtain the electricity demand function $q(p; \theta)$ and the inverse demand function $p(q; \theta)$ as follows:

$$q(p; \theta) = \frac{a\theta - p}{b\theta^2}, \quad p(q; \theta) = a\theta - b\theta^2 q \tag{5}$$

[3] By considering $u(s)$ as profit, we can interpret this problem as a producer's problem. In that case, s is the amount of production, and θ is the energy efficiency in production.

2.2 Market clearing

For simplicity, consider that the supply-side resources in the day-ahead market consist of two power sources. One is variable renewable energy (solar, wind, etc.), and the other is a nonvariable power generation resource (hydro, nuclear, etc.). The predicted supply of variable renewable energy q_R ($q_R > 0$) is exogenously determined. We discard the capacity constraint of the nonvariable generation plant. Suppose that the marginal cost of variable renewable energy c_R and the marginal cost of nonvariable sources c_0 are both constant. We also assume that the marginal cost of variable renewable energy is less than or equal to that of nonvariable generation ($c_0 \geq c_R$). This assumption ensures that variable renewable energy is prioritized in the merit order. The emission factor[4] for the variable renewable energy source is zero, and the emission factor for the nonvariable generation resource α is greater than or equal to zero ($\alpha \geq 0$).[5]

Suppose that a carbon tax is imposed on CO_2 emissions. Multiplying the carbon tax rate t (the tax imposed per 1 kg-CO_2) by the emission factor of the power source yields the tax per 1 kWh of electricity generated. Thus, the marginal cost of the nonvariable power source, including the carbon tax, is represented by the following:

$$mc_0 = c_0 + \alpha t \tag{6}$$

The following discussion focuses on a case where the nonvariable power source is a marginal plant.[6] Since the marginal plant is the first plant to react to changes in demand, its technical characteristics are particularly important when examining the effects of changes in demand (Germeshausen and Wölfing, 2020). For this reason, for most of this paper, we can simplify the explanation without losing generality by focusing on two types of power sources—variable renewable energy and a nonvariable marginal plant—and discarding the others. To ensure that the nonvariable power plant is the marginal plant, we assume the following condition in addition to discarding

[4]The emission factor here represents CO_2 emissions per 1 kWh of electricity generated (kg-CO_2/kWh).

[5]Nonvariable power sources here can also include zero-emission power sources such as hydro and nuclear power. These zero-emission generation resources may become marginal plants when decarbonizing electricity toward carbon neutrality.

[6]A marginal plant refers to the plant with the highest marginal cost, positioned at the end of the merit order, among the plants operating to meet demand.

the capacity constraint.[7]

$$q_R < \frac{a\theta - (c_0 + \alpha t)}{b\theta^2} \tag{7}$$

The market clearing price p_0 in the day-ahead market is determined by the regulator to be equal to the marginal cost of the marginal plant:

$$p_0 = c_0 + \alpha t \tag{8}$$

By substituting Eq. (8) into the demand function shown in Eq. (5), we obtain the market clearing quantity of the day-ahead market, which is denoted by q_0, as follows:[8]

$$q_0 \equiv q(p_0; \theta) = \frac{a\theta - (c_0 + \alpha t)}{b\theta^2} \tag{9}$$

The equilibrium quantity q_0 in the day-ahead market is the baseline for determining the quantity of DR in the balancing market. The remaining quantity obtained by subtracting the supply of variable renewable energy q_R from the equilibrium quantity q_0 in the day-ahead market is the supply of marginal (nonvariable) plant q_{NV}.

$$q_{NV} = q_0 - q_R \tag{10}$$

The supply of the marginal plant is fixed at the time the day-ahead market closes. To cope with subsequent (after the closure of the day-ahead market) fluctuations in the renewable energy supply, other, more responsive sources (thermal power) are used in the balancing market.

[7]When the consumer's marginal utility at the supply level of a variable renewable source exceeds the marginal cost of a nonvariable generation plant, the optimal choice is to operate the nonvariable generation source alongside the variable renewable energy source. In this case, the nonvariable plant becomes the marginal plant. The marginal utility at the supply level q_R is determined by substituting q_R into the inverse demand function described in Eq. (5). The marginal cost of the nonvariable power source is represented in Eq. (6). Furthermore, as previously mentioned, this analysis does not consider the capacity constraint of the nonvariable generation resource. Therefore, the condition for the nonvariable generation resource to operate as a marginal plant can be expressed as follows:

$$a\theta - b\theta^2 q_R > c_0 + \alpha t$$

From this, we obtain Eq. (7).

[8]The equilibrium quantity q_0 in the day-ahead market is a function of the energy efficiency θ and the carbon tax rate t: $q_0 = q_0(\theta, t)$.

3 Balancing market

After the equilibrium (baseline) of the day-ahead market is established, a shortage of $x \in [0, q_R]$ occurs in the supply of variable renewable energy. The role of the balancing market is to restore the supply–demand balance. In the balancing market, the system operator is responsible for restoring the supply–demand balance given the observed value of x. Suppose that thermal power generation and incentive-based DR are available to restore the supply–demand balance. The system operator, as a social planner, determines the amount of thermal power generation q_f and DR q_d to minimize the social cost of balancing.

3.1 Cost of DR

In incentive-based DR, consumers receive payments for demand reduction on the basis of the baseline q_0 set in the day-ahead market. By letting q_d be the quantity of demand reduction and r be the reward per unit of demand reduction, the reward received by the consumer (demand-side aggregator) is rq_d.

Let us first formulate the costs of DR to consider consumer decision-making in the balancing market. The potential utility of energy services lost by demand reduction corresponds to the cost of DR. From Eq. (5), we obtain the DR cost function $C_d(q_d; \theta)$ and the marginal cost function $mc_d(q_d; \theta)$ as follows:

$$C_d(q_d; \theta) \equiv \int_{q_0 - q_d}^{q_0} p(q; \theta)\ dq = p_0 q_d + \frac{b\theta^2}{2} q_d^2, \tag{11}$$

$$mc_d(q_d; \theta) \equiv \frac{\partial C_d(q_d; \theta)}{\partial q_d} = p_0 + b\theta^2 q_d. \tag{12}$$

From Eq. (12), we obtain the following proposition:

Proposition 1. An improvement in energy efficiency (an increase in θ) increases the marginal cost of DR.

$$\frac{\partial mc_d(q_d; \theta)}{d\theta} = 2b\theta q_d > 0, \quad \forall q_d > 0. \tag{13}$$

Improved energy efficiency increases the amount of energy service that one unit of electricity produces, thus increasing the potential utility lost by reducing electricity demand and increasing the marginal cost of DR.

3.2 Consumer decision-making

Next, let us investigate the problem of demand reduction by the consumer. Given that the baseline demand is q_0 and the amount of demand reduction in the DR is q_d, the final (actual) amount of electricity consumed by the consumer is $q_0 - q_d$.

The consumer surplus in the balancing market, which is denoted by CS_1, is the sum of the utility produced by the electricity finally consumed and the reward received in DR minus the expenditure to purchase the electricity, as represented by the following expression:

$$CS_1 \equiv \int_0^{q_0 - q_d} p(q;\theta)\ dq - p_0(q_0 - q_d) + rq_d. \tag{14}$$

From Eq. (11), we can rewrite Eq. (14) as follows:

$$CS_1(q_d;\theta) = CS_0 - C_d(q_d;\theta) + (p_0 + r)q_d. \tag{15}$$

where CS_0 is the consumer surplus in the day-ahead market:

$$CS_0 \equiv \int_0^{q_0} p(q;\theta)\ qd - p_0 q_0. \tag{16}$$

Since CS_0 is fixed with the closure of the day-ahead market, it does not affect consumer decision making in the balancing market.

The consumer chooses the amount of demand reduction q_d such that the consumer surplus CS_1 represented in Eq. (15) is maximized. The first-order condition for this problem is as follows:

$$\frac{\partial C_d(q_d;\theta)}{\partial q_d} = p_0 + r. \tag{17}$$

The left-hand side and the right-hand side of Eq. (17) are the marginal cost and marginal revenue of DR, respectively.[9] By substituting Eq. (12) into Eq. (17) and rearranging terms, we have the supply function and the inverse supply function of DR:

$$q_d(r;\theta) = \frac{r}{b\theta^2}, \quad r(q_d;\theta) = b\theta^2 q_d. \tag{18}$$

[9]Note that the marginal revenue of DR is $p_0 + r$, not r. For each unit of demand reduction, the consumer saves the payment to purchase electricity, which is equivalent to the price p_0, adding to receiving a reward r.

3.3 Thermal power in the balancing market

Suppose that the cost of thermal power generation in the balancing market is represented as a quadratic function as follows:

$$C_f(q_f) = c_f q_f + \frac{b_f}{2} q_f^2 + \beta t q_f, \tag{19}$$

where β and q_f are the emission factor and the amount of thermal power generation in the balancing market, respectively. From Eq. (19), we obtain the marginal cost of thermal power generation:

$$mc_f(q_f) = (c_f + \beta t) + b_f q_f \tag{20}$$

Here, we make the following assumption:

Assumption 1.

1. $c_f > c_0$. This ensures that the marginal cost of thermal power generation in the balancing market is greater than the marginal cost of the marginal plant in the day-ahead market.
2. $\beta > \alpha$. This means that the emission factor of thermal power generation in the balancing market is greater than the emission factor of the marginal plant in the day-ahead market.

The thermal power producer determines its output q_f to maximize the profit while taking the electricity price in the balancing market (which is denoted by p_f) as given.

$$\underset{q_f}{\text{maximize}}\ p_f q_f - C_f(q_f) \tag{21}$$

The first-order condition for the problem of Eq. (21) is as follows:

$$(c_f + \beta t) + b_f q_f = p_f \tag{22}$$

From Eq. (22), we obtain the supply function and inverse supply function of thermal power generation in the balancing market:

$$q_f(p_f) = \frac{-(c_f + \beta t) + p_f}{b_f}, \quad p_f(q_f) = (c_f + \beta t) + b_f q_f. \tag{23}$$

3.4 System operations

In the balancing market, the system operator is responsible for restoring the supply–demand balance, given the observed value of shortfall x due to variable renewable energy.

As a social planner, the system operator aims to minimize social costs to restore the supply-demand balance. The social cost of DR is not the reward paid for demand reduction but the loss of potential benefit, which is represented by Eq. (11). Regarding the social cost of thermal power generation, the system operator does not have the ability to assess the marginal external cost of the emissions of carbon dioxide; instead, it considers the tax rate t set by the government to be the marginal external cost. Therefore, the cost shown in Eq. (19) is considered the social cost of thermal power generation here. The problem for the system operator is written as follows:

$$\begin{aligned} \underset{q_d, q_f}{\text{minimize}} \quad & C_d(q_d) + C_f(q_f) \\ \text{subject to} \quad & q_d + q_f = x, \\ & q_d \geq 0,\ q_f \geq 0. \end{aligned} \tag{24}$$

The solutions for this problem are as follows:[10] (In the following, equilibrium solutions for balancing markets are denoted by superscript asterisks.)

$$q_d^* = q_d^*(x, \theta, t) = \begin{cases} x & (0 \leq x \leq \underline{x}) \\ \dfrac{(c_f + \beta t) + b_f x - (c_0 + \alpha t)}{b\theta^2 + b_f} & (\underline{x} < x \leq q_R) \end{cases}, \tag{25}$$

$$q_f^* = q_f^*(x, \theta, t) = \begin{cases} 0 & (0 \leq x \leq \underline{x}) \\ \dfrac{b\theta^2 x - [(c_f + \beta t) - (c_0 + \alpha t)]}{b\theta^2 + b_f} & (\underline{x} < x \leq q_R) \end{cases}, \tag{26}$$

where $\underline{x}$ is defined as a function of θ and t as follows:

$$\underline{x} = \underline{x}(\theta, t) = \frac{(c_f + \beta t) - (c_0 + \alpha t)}{b\theta^2} > 0. \tag{27}$$

Eqs. (25) and (26) show that if the shortfall x is smaller than $\underline{x}$, the optimal choice for the system operator is to restore the supply–demand balance using

[10] See Appendix 3 for the derivation.

only DR without using thermal power generation. On the other hand, if x is greater than $\underline{x}$, the cost-effective choice for the system operator is to use thermal power generation in addition to DR.

By differentiating Eqs. (26) and (27) by θ, we obtain the following proposition;

Proposition 2.

1. For any $x \in [\underline{x}, q_R]$, improved energy efficiency decreases the amount of DR and increases the generation of thermal power in the balancing market. That is,

$$\frac{\partial q_d^*}{\partial \theta} = \frac{-2b\theta}{(b\theta^2 + b_f)} \left[\frac{b_f x - (c_0 + \alpha t) + (c_f + \beta t)}{(b\theta^2 + b_f)} \right] < 0, \quad \forall x \in [\underline{x}, q_R], \tag{28}$$

$$\frac{\partial q_f^*}{\partial \theta} = \frac{2b\theta}{(b\theta^2 + b_f)} \left[\frac{b_f x - (c_0 + \alpha t) + (c_f + \beta t)}{(b\theta^2 + b_f)} \right] > 0, \quad \forall x \in [\underline{x}, q_R]. \tag{29}$$

2. The improved energy efficiency extends the range of x over which thermal power generation operates in the balancing market. That is,

$$\frac{\partial \underline{x}}{\partial \theta} < 0. \tag{30}$$

Proposition 2 shows that improved energy efficiency increases the marginal cost of DR (Proposition 1), thereby increasing the reliance on thermal power generation in the balancing market.

4 Impact of energy efficiency

4.1 Impact on the day-ahead market

To examine the impact of improved energy efficiency (an increase in θ) on the equilibrium quantity of the day-ahead market q_0, partially differentiating Eq. (9) by θ yields

$$\frac{\partial q_0}{\partial \theta} = \frac{-a}{b\theta^2} \left(\frac{1 - |\varepsilon_0|}{1 + |\varepsilon_0|} \right) \begin{cases} < 0 & (|\varepsilon_0| < 1) \\ = 0 & (|\varepsilon_0| = 1) \\ > 0 & (|\varepsilon_0| > 1) \end{cases}, \tag{31}$$

where ε_0 is the price elasticity of demand at market price p_0.[11]

From Eq. (31), we can see that an increase in energy efficiency decreases the equilibrium quantity of the day-ahead market if (and only if) the demand is inelastic at price p_0 (the absolute value of ε_0 is less than 1). Conversely, an increase in energy efficiency increases the equilibrium quantity in the day-ahead market if the demand is elastic at the market price (the absolute value of ε_0 is greater than 1).[12] In the following sections, we restrict our discussion to a case where energy efficiency improvements reduce demand in the day-ahead market. For this purpose, we assume that the following condition holds:

Assumption 2. In the day-ahead market, the absolute value of the price elasticity of demand ε_0 at market price p_0 is less than 1:

$$|\varepsilon_0| < 1 \Leftrightarrow a\theta > 2(c_0 + \alpha t). \tag{32}$$

From Eqs. (8) and (A.3), the condition $a\theta > 2(c_0 + \alpha t)$ shown in Eq. (32) is equivalent to the absolute value of ε_0 being less than 1.

Next, we examine the impact of energy efficiency on the supply of the marginal plant. Note that q_R is exogenously given, and partially differentiating Eq. (10) by θ yields

$$\frac{\partial q_{NV}}{\partial \theta} = \frac{\partial q_0}{\partial \theta} < 0. \tag{33}$$

From Eq. (31) and assumption 2, the sign of Eq. (33) is negative. This means that improved energy efficiency decreases the supply of the marginal plant in the day-ahead market.

The emission factor α of the marginal plant in the day-ahead market multiplied by the amount of electricity it generates, as shown in Eq. (10), is the amount of emission in the day-ahead market, which is denoted by E_0:

$$E_0 = E_0(\theta, t) = \alpha q_{NV}. \tag{34}$$

[11]See Appendix 1 for the derivation of Eq. (31).

[12]This situation arises from the energy rebound effect. However, since the impact of the energy rebound effect is not the main focus of this paper, we assume that it is limited and that demand decreases due to improvements in energy efficiency. See Appendix 2 for a formulation of the energy rebound effect in this model.

To examine the effects of energy efficiency on the emissions of the day-ahead market, partially differentiating Eq. (34) by θ yields,

$$\frac{\partial E_0}{\partial \theta} = \alpha \frac{\partial q_{NV}}{\partial \theta}. \tag{35}$$

From Eqs. (33) and (35), we have

$$\frac{\partial E_0}{\partial \theta} \leq 0, \tag{36}$$

where the equality holds only if $\alpha = 0$. As Eq. (36) shows, improved energy efficiency decreases the emissions of the day-ahead market if the emission factor of the marginal plant (α) is greater than zero, whereas it does not affect the emissions of the day-ahead market when $alpha = 0$.

4.2 Impact on the balancing market

In this section, we demonstrate how demand-side energy efficiency affects emissions in the balancing market. In formulating the emissions of the balancing market, we consider that the shortfall $x \in [0, q_R]$ follows a probability density function $f(x)$.[13] Here, we assume that the value of $f(x)$ is greater than (not equal to) zero for any x in the domain $0 \leq x \leq q_R$.

Assumption 3. $f(x) > 0$ for any $x \in [0, q_R]$.

The amount of thermal power generation for any x, which is represented as $q_f^*(x, \theta, t)$, is shown by Eq. (26). Let the emission factor of the thermal power generation in the balancing market be β, as mentioned above. We formulate the emissions in the balancing market E_1 as follows:

$$E_1 = E_1(\theta, t) = \int_0^{q_R} \beta q_f^*(x, \theta, t) f(x)\ dx. \tag{37}$$

From Eq. (26), $q_f^* = 0$ for any $x \in [0, \underline{x}]$. Therefore, we can rewrite Eq. (37) as follows:

$$E_1 = E_1(\theta, t) = \int_{\underline{x}(\theta,t)}^{q_R} \beta q_f^*(x, \theta, t) f(x)\ dx. \tag{38}$$

Here, we obtain the following proposition regarding the impact of energy efficiency on emissions in the balancing market:

[13]Note, however, that the system operator's problem is based on the observed value of x, as represented in Eq. (24).

Proposition 3. Improved energy efficiency increases emissions in the balancing market. That is,

$$\frac{\partial E_1}{\partial \theta} > 0. \tag{39}$$

Proof. By partially differentiating Eq. (38) by θ, we have

$$\frac{\partial E_1}{\partial \theta} = \int_{\underline{x}(\theta,t)}^{q_R} \beta \frac{\partial q_f^*(x,\theta,t)}{\partial \theta} f(x)\ dx - \beta q_f^*(\underline{x},\theta,t) f(\underline{x}) \frac{\partial \underline{x}(\theta,t)}{\partial \theta}. \tag{40}$$

Because $q_f^*(\underline{x},\theta,t) = 0$ from Eq. (26), we can rewrite Eq. (40) as follows:

$$\frac{\partial E_1}{\partial \theta} = \int_{\underline{x}(\theta,t)}^{q_R} \beta \frac{\partial q_f^*(x,\theta,t)}{\partial \theta} f(x)\ dx. \tag{41}$$

From Assumptions 1 and 3 and Eq. (29), the value of the integrand in Eq. (41) is greater than zero in the interval from $\underline{x}$ to q_R. Therefore, $\partial E_1/\partial\theta > 0$. □

Proposition 3 shows that improved energy efficiency increases the marginal cost of DR (Proposition 1), which leads to an increase in the amount of thermal power generation (Proposition 2) and results in an increase in emissions in the balancing market.

4.3 Impact on total emissions

We define the total emissions E as the sum of the emissions of the day-ahead market (E_0) and the balancing market (E_1):

$$E \equiv E_0 + E_1 \tag{42}$$

The impact of energy efficiency θ on total emissions is given by

$$\frac{\partial E}{\partial \theta} = \frac{\partial E_0}{\partial \theta} + \frac{\partial E_1}{\partial \theta}, \tag{43}$$

where the impacts on the day-ahead market ($\partial E_0/\partial\theta$) and the balancing market ($\partial E_1/\partial\theta$) are shown in Eqs. (36) and (39), respectively.

If the emission factor of the marginal plant in the day-ahead market is sufficiently small, then the effect of improved energy efficiency on reducing emissions in the day-ahead market becomes negligible, and only the effect of increasing emissions in the balancing market (which is represented by proposition 3) matters. In that case, an improvement in energy efficiency increases total emissions. Then, we obtain the following proposition:

Proposition 4. When the emission factor of the marginal plant in the day-ahead market is sufficiently small, an improvement in energy efficiency increases total emissions. That is,

$$\lim_{\alpha \to 0} \frac{\partial E}{\partial \theta} > 0. \tag{44}$$

Proof. Let us show that Proposition 4 holds by showing that $\lim_{\alpha \to 0} \frac{\partial E_0}{\partial \theta} = 0$ and $\lim_{\alpha \to 0} \frac{\partial E_1}{\partial \theta} > 0$, respectively.

(i) Proof for $\lim_{\alpha \to 0} \frac{\partial E_0}{\partial \theta} = 0$
$\frac{\partial E_0}{\partial \theta} = \alpha \frac{\partial q_0}{\partial \theta}$ from Eqs. (33) and (35), and $|\frac{\partial q_0}{\partial \theta}| < \frac{a}{b\theta^2}$ from Eq. (31). Then $\lim_{\alpha \to 0} \frac{\partial E_0}{\partial \theta} = 0$ holds.

(ii) Proof for $\lim_{\alpha \to 0} \frac{\partial E_1}{\partial \theta} > 0$
From Eq. (29),

$$\lim_{\alpha \to 0} \frac{\partial q_f^*}{\partial \theta} = \frac{2b\theta}{(b\theta^2 + b_f)} \left[\frac{b_f x + (c_f + \beta t) - c_0}{(b\theta^2 + b_f)} \right] > 0. \tag{45}$$

From Eq. (41),

$$\lim_{\alpha \to 0} \frac{\partial E_1}{\partial \theta} = \int_{\underline{x}(\theta,t)}^{q_R} \beta \left(\lim_{\alpha \to 0} \frac{\partial q_f^*}{\partial \theta} \right) f(x)\, dx. \tag{46}$$

From Assumptions 2 and 3 and Eq. (45), the value of the integrand in Eq. (46) is greater than zero in the interval from $\underline{x}$ to q_R. Therefore, $\lim_{\alpha \to 0} \frac{\partial E_1}{\partial \theta} > 0$ holds.

From (i), (ii) and Eq. (43),

$$\lim_{\alpha \to 0} \frac{\partial E}{\partial \theta} = \lim_{\alpha \to 0} \frac{\partial E_0}{\partial \theta} + \lim_{\alpha \to 0} \frac{\partial E_1}{\partial \theta} > 0.$$

□

5 Energy efficiency and carbon tax

This section examines the combination of energy efficiency and carbon taxes to achieve a given target level of CO_2 emissions. Suppose that the government cannot estimate the marginal external cost of CO_2 emissions, nor does it set the carbon tax rate on the basis of it. Instead, the government sets a target for total CO_2 emissions in advance, denoted by $\bar{E}$, and sets the carbon tax rate t to meet that target, considering the energy efficiency level θ. In the following, we examine the impact of energy efficiency θ on the carbon tax rate t under an exogenously determined total CO_2 emissions target $\bar{E}$.

By totally differentiating Eq. (42) with respect to θ, we obtain

$$dE = \frac{\partial E}{\partial \theta} d\theta + \frac{\partial E}{\partial t} dt. \tag{47}$$

To examine the combinations of energy efficiency levels and carbon tax rates that would result in total emissions of exactly $\bar{E}$, substituting $dE = 0$ into Eq. (47) and rearranging the terms yields,

$$\frac{dt}{d\theta} = -\frac{\partial E/\partial \theta}{\partial E/\partial t}. \tag{48}$$

Here, the denominator of the right-hand side of Eq. (48) is negative:[14]

$$\frac{\partial E}{\partial t} < 0. \tag{49}$$

From Eqs. (48) and (49), we have

$$\frac{dt}{d\theta} = \begin{cases} < 0 & (\frac{\partial E}{\partial \theta} < 0), \\ = 0 & (\frac{\partial E}{\partial \theta} = 0), \\ > 0 & (\frac{\partial E}{\partial \theta} > 0). \end{cases} \tag{50}$$

From Eq. (50) and Proposition 4, we obtain the following proposition:

Proposition 5. When the emission factor of the marginal plant in the day-ahead market (denoted by α) is sufficiently small, improved energy efficiency (an increase in θ) will increase the carbon tax rate needed to achieve the emissions target $\bar{E}$. That is,

$$\lim_{\alpha \to 0} \frac{dt}{d\theta} > 0. \tag{51}$$

[14] See Appendix 4 for the derivation of Eq. (49).

Improved energy efficiency reduces emissions in the day-ahead market while increasing emissions in the balancing market, as shown in Eq. (36) and Proposition 3, respectively. This leads to two possible cases regarding the impact of improved energy efficiency on total emissions. If the day-ahead market is the primary source of emissions, total emissions will decrease due to improved energy efficiency. On the other hand, if the balancing market is the primary source of emissions, total emissions increase due to improved energy efficiency. In the first case, the policy target $\bar{E}$ can be achieved with a lower carbon tax rate, whereas in the second case, the carbon tax rate must be increased to offset the increase in total emissions.

6 Conclusions

We have shown that improved energy use efficiency on the demand side increases the marginal cost of DR, which in turn increases the amount of thermal power generation in a balancing market. In particular, if the emission factor of a marginal plant in a day-ahead market is sufficiently small, only the emissions of the balancing market matter, and improved energy efficiency could increase total emissions through a mechanism other than the energy rebound effect. In such cases, the carbon tax rate would need to be increased to offset the increase in thermal power generation due to improved energy efficiency by promoting DR.

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Appendix 1 Derivation of Eq. (31)

Differentiating Eq. (9) by θ, we have

$$\frac{dq_0}{d\theta} = \frac{2p_0 - a\theta}{b\theta^3}. \tag{A.1}$$

The price elasticity of demand ε is represented as a function of price p and energy efficiency θ as follows.

$$\varepsilon(p, \theta) \equiv \frac{p}{q(p;\theta)} \frac{\partial q(p;\theta)}{\partial p} \tag{A.2}$$

Substituting Eqs. (5) and (8) into (A.2), we obtain the price elasticity of demand at the market price p_0:

$$\varepsilon_0 \equiv \varepsilon(p_0; \theta) = \frac{-p_0}{a\theta - p_0}. \tag{A.3}$$

From Eq. (A.3),

$$p_0 = a\theta \left(\frac{|\varepsilon_0|}{1 + |\varepsilon_0|} \right). \tag{A.4}$$

Substituting Eq. (A.4) into Eq. (A.1) yields Eq. (31).

Appendix 2 Energy rebound effect

Let us formulate the rebound effect in this model. The consumer's problem in Eq. (3) is equivalent to the problem of choosing an amount of energy service s instead of electricity q, as shown below:

$$\underset{s}{\text{maximize}} \left(as - \frac{b}{2}s^2 \right) - \frac{p}{\theta}s. \tag{A.5}$$

The first-order condition for the problem of Eq. (A.5) is as follows:

$$a - bs = \frac{p}{\theta}. \tag{A.6}$$

The left-hand side of Eq. (A.6) is the marginal utility of energy service, and the right-hand side is the price of energy service. Improved energy efficiency lowers the price of energy services and increases consumer demand for energy services. By differentiating both sides of Eq. (A.6) by θ and rearranging the terms, we obtain

$$\frac{ds}{d\theta} = \frac{p}{b\theta^2} > 0. \tag{A.7}$$

Here, since $s \equiv \theta q$, we have

$$\frac{ds}{d\theta} = q + \theta \frac{dq}{d\theta}. \tag{A.8}$$

From Eq. (A.8), the impact of energy efficiency on electricity demand can be represented as a combined effect of positive and negative effects, as shown in the following equation:

$$\frac{dq}{d\theta} = \underbrace{\frac{1}{\theta}\frac{ds}{d\theta}}_{+} \underbrace{- \frac{q}{\theta}}_{-}. \tag{A.9}$$

The positive effect on the right-hand side of Eq. (A.9) can be interpreted as the energy rebound effect in this model. By substituting Eqs. (9) and (A.7) into Eq. (A.9), we obtain Eq. (A.1).

Appendix 3 Solution for Eq. (24)

The Lagrangian function for the problem in Eq. (24) is as follows:

$$\begin{aligned} L(q_d, q_f, \lambda, \mu_1, \mu_2) = C_d(q_d) + C_f(q_f) + \lambda(x - q_d - q_f) \\ -\mu_1 q_d - \mu_2 q_f. \end{aligned} \tag{A.10}$$

The KKT (Karush–Kuhn–Tucker) conditions are as follows:

$$C'_d(q_d) = \lambda + \mu_1 \tag{A.11}$$
$$C'_f(q_f) = \lambda + \mu_2 \tag{A.12}$$
$$x - q_d - q_f = 0 \tag{A.13}$$
$$q_d \geq 0,\ \mu_1 \geq 0,\ \mu_1 q_d = 0 \tag{A.14}$$
$$q_f \geq 0,\ \mu_2 \geq 0,\ \mu_2 q_f = 0 \tag{A.15}$$

(i) If $q_d = 0$, $q_f = x$ from Eq. (A.13). From Eqs. (A.14) and (A.15),

$$\mu_1 \geq 0, \quad \mu_2 = 0.$$

From Eqs. (A.11) and (A.12), we have

$$C'_d(0) = \lambda + \mu_1, \quad C'_f(x) = \lambda,$$

where $C'_d(0) \geq C'_f(x)$ from $\mu_1 \geq 0$. Substituting Eqs. (12) and (20) yields

$$p_0 \geq (c_f + \beta t) + b_f x.$$

However, this contradicts Assumption 1. Thus, this case ($q_d = 0$) is excluded from the candidate solutions.

(ii) If $q_d > 0$ and $q_f > 0$, from Eqs. (A.14) and (A.15), we have

$$\mu_1 = \mu_2 = 0.$$

Substituting Eqs. (12), (20), and $\mu_1 = \mu_2 = 0$ into Eqs. (A.11) and (A.12), we obtain

$$(c_0 + \alpha t) + b\theta^2 q_d = (c_f + \beta t) + b_f q_f = \lambda. \tag{A.16}$$

Eq. (A.16) implies that q_d and q_f are determined so that the marginal costs of DR and thermal power generation are equal. From Eqs. (A.13) and (A.16), we obtain

$$\begin{aligned} q_d^* &= \frac{(c_f + \beta t) + b_f x - (c_0 + \alpha t)}{b\theta^2 + b_f}, \\ q_f^* &= \frac{b\theta^2 x - [(c_f + \beta t) - (c_0 + \alpha t)]}{b\theta^2 + b_f}, \\ \lambda^* &= \frac{b_f(c_0 + \alpha t) + b\theta^2(c_f + \beta t) + b\theta^2 b_f x}{b\theta^2 + b_f}, \end{aligned}$$

where $q_d^* > 0$ is secured by Assumption 1. Furthermore, $q_f^* > 0$ is equivalent to the following condition:

$$x > \frac{(c_f + \beta t) - (c_0 + \alpha t)}{b\theta^2}.$$

The DR and thermal power generation prices, denoted by r^* and p_f^*, are obtained from Eqs. (18), (23) and (A.16) as follows:

$$r^* = \lambda^* - p_0, \quad p_f^* = \lambda^*.$$

(iii) If $q_d = x$, then $q_f = 0$ from Eq. (A.13). From Eqs. (A.14) and (A.15), we have

$$\mu_1 = 0, \quad \mu_2 \geq 0.$$

Substituting $q_d^* = x$ and $q_f^* = 0$ into Eqs. (A.11) and (A.12) yields

$$C_d'(x) = \lambda, \quad C_f'(0) = \lambda + \mu_2,$$

where $C_d'(x) \leq C_f'(0)$ since $\mu_2 \geq 0$. Substituting Eqs. (12) and (20) yields

$$\lambda^* = (c_0 + \alpha t) + b\theta^2 x \leq c_f + \beta t.$$

Then, we have

$$x \leq \frac{(c_f + \beta t) - (c_0 + \alpha t)}{b\theta^2}.$$

Substituting $q_d^* = x$ into Eq. (18), we obtain the DR price as follows:

$$r^* = b\theta^2 x.$$

From (ii) and (iii), we obtain Eqs. (25) and (26).

Appendix 4 Derivation of Eq. (49)

First, we look at the impact of the carbon tax on emissions in the day-ahead market. By substituting Eqs. (9) and (10) into Eq. (34) and partially differentiating it with t, we obtain

$$\frac{\partial E_0}{\partial t} = \frac{-\alpha^2}{b\theta^2} \leq 0. \tag{A.17}$$

Next, to examine the impact of the carbon tax on emissions in the balancing market, partially differentiating Eq. (38) with t yields

$$\frac{\partial E_1}{\partial t} = \int_{\underline{x}(\theta,t)}^{q_R} \beta \frac{\partial q_f^*(x,\theta,t)}{\partial t} f(x)\ dx - \beta q_f^*(\underline{x},\theta,t) f(\underline{x}) \frac{\partial \underline{x}(\theta,t)}{\partial t} \tag{A.18}$$

Since $q_f^*(\underline{x},\theta,t) = 0$ from Eq. (26), Eq. (A.18) can be rewritten as

$$\frac{\partial E_1}{\partial t} = \int_{\underline{x}(\theta,t)}^{q_R} \beta \frac{\partial q_f^*(x,\theta,t)}{\partial t} f(x)\ dx. \tag{A.19}$$

Here, partially differentiating Eq. (26) with t yields

$$\frac{\partial q_f^*(x,\theta,t)}{\partial t} = \frac{-(\beta-\alpha)}{b\theta^2 + b_f}. \tag{A.20}$$

From Assumption 2 ($\beta > \alpha$), the value of Eq. (A.20) is negative. From Assumptions 1 and 3 and Eq. (A.20), the value of the integrand in Eq. (A.19) is less than zero in the interval from $\underline{x}$ to q_R. Therefore,

$$\frac{\partial E_1}{\partial t} < 0. \tag{A.21}$$

From Eqs. (42), (A.17), and (A.21), we have

$$\frac{\partial E}{\partial t} = \frac{\partial E_0}{\partial t} + \frac{\partial E_1}{\partial t} < 0.$$

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